Outline

- Applied General Equilibrium Modeling
- MCP Formulation
- MPSGE Application

Applied General Equilibrium Modeling

Two Alternative Formats

- System of equations
  - directly write first order/optimality conditions
  - tedious for large models
- Mixed Complementarity Format (MCP)
  - quantities and prices as variables
  - prices are complementary to balance equations
  - Find $z \in R$ s.t. $f(z) \geq 0$, $z \geq 0$ and $z' f(z)=0$
  - MPSGE available as tool for specification of MCP model

Applied General Equilibrium Modeling

Equilibrium Conditions

1. **Zero profit** : cost of production $\geq$ revenue
2. **Market clearance** : supply $\geq$ demand
3. **Income balance** : factor income $\geq$ expenditure

$\rightarrow$ *unique set of equilibrium prices*

Applied General Equilibrium Modeling

Example: *2-good, 2-factor closed economy with fixed factor endowments, one representative consumer.*

(1) $X = X(L_x , K_x)$

(2) $Y = Y(L_y , K_y)$

(3) $L = L_x + L_y$

(4) $K = K_x + K_y$

(5) $W = W(X, Y)$

(6) $I = p_L L + p_K K = p_x X + p_y Y$

Applied General Equilibrium Modeling

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Equilibrium could be solved for by a constrained optimization problem:

Max. (5)

Subject to (1), (2), (3), (4), and (6)

What if there are two different consumer types with different preferences and different factor endowments ?

What do we maximize?

The usefulness of this approach breaks down quickly as the model becomes more complicated.

Alternative approach: convert the problem to a system of equations, and solve that system.

Applied General Equilibrium Modeling

Converting the problem to a system of equations:
First, solve the underlying cost minimization problems for **producers** and **consumers**, *so that individual optimizing behavior is embedded in the model.*

Producers: we want to solve for cost functions for each sector, which give the minimum cost of producing a good at given factor prices.

Unit cost functions for X and Y: $c_X = c_X(p_L, p_K)$, $c_Y = c_Y(p_L, p_K)$

Consumers: we want to solve for a cost function for the consumer, commonly called an expenditure function, which gives the minimum cost at given commodity prices of buying one unit of utility

Unit cost (expenditure) function for the Consumer: $e = e(p_X, p_Y)$

Assume the general form of a two-input Cobb-Douglas production function

$X = a\, L^{\alpha}\, K^{\beta}$

It can be shown that the minimum cost for x is

$$C_X(p_L, p_K, x) = a^{-1/(\alpha+\beta)} \left[(\alpha/\beta)^{\beta/(\alpha+\beta)} + (\alpha/\beta)^{-\alpha/(\alpha+\beta)}\right] (p_L^{\alpha/(\alpha+\beta)})\, (p_K^{\beta/(\alpha+\beta)})\, (x^{1/(\alpha+\beta)})$$

if $\alpha+\beta \neq 1$

$C_X(p_L, p_K, x) = p_L^{\alpha}\; p_K^{\beta}\; x$ if $\alpha+\beta = 1$

Shephard's Lemma: the demand for a particular good *i* for a given level of utility *u* and given prices **p**, equals the derivative of the expenditure function with respect to the price of the relevant good

$$\frac{\partial c_X}{\partial p_L} = dl_X = \text{X producer's demand for labor per unit of output}$$

$$\frac{\partial c_Y}{\partial p_L} = dl_Y = \text{Y producer's demand for labor per unit of output}$$

$$\frac{\partial c_X}{\partial p_K} = dk_X = \text{X producer's demand for capital per unit of output}$$

$$\frac{\partial c_Y}{\partial p_L} = dk_Y = \text{Y producer's demand for capital per unit of output}$$

$$\frac{\partial e}{\partial p_X} = dx_U = \text{Consumer's demand for X per unit of utility}$$

Applied General Equilibrium Modeling

(1) Zero profit for X	$c_X(p_L, p_K) \geq p_X$	
(2) Zero profit for Y	$c_Y(p_L, p_K) \geq p_Y$	**Zero profit**
(3) Zero profit for W	$e(p_X, p_Y) \geq p_W$	
(4) Market clearance for X	$p_X\, X \geq e(p_X, p_Y)\, W$	
(5) Market clearance for Y	$p_Y\, Y \geq e(p_X, p_Y)\, W$	
(6) Market clearance for W	$p_W\, W \geq I$	**Market clearance**
(7) Market clearance for L	$L \geq dl_X\, X + dl_Y\, Y$	
(8) Market clearance for K	$K \geq dk_X\, X + dk_Y\, Y$	
(9) Income balance	$I = p_L L + p_K K$	**Income balance**

Applied General Equilibrium Modeling

Formulating equilibrium as a complementarity problem requires that each inequality is associated with a particular variable.

Think about the economics of what must be true if a particular weak inequality holds as a strict inequality.

If a zero profit conditions holds as a strict inequality in equilibrium, profits for that activity are negative, so that good will not be produced. Thus the complementary variable to a zero-profit condition is a quantity, the activity level.

If a market-clearing condition holds as a strict inequality, supply exceeds demand for that good or factor in equilibrium so its price must be zero. Thus the complementary variable to a market clearing equation is the price of that good or factor.

The complementary variable to an income balance equation is just the income of that agent.

Applied General Equilibrium Modeling

		Complementary Variable
(1) Zero profit for X	$c_X(p_L, p_K) \geq p_X$	X
(2) Zero profit for Y	$c_Y(p_L, p_K) \geq p_Y$	Y
(3) Zero profit for W	$e(p_X, p_Y) \geq p_W$	W
(4) Market clearance for X	$X \geq e_{p_X}(p_X, p_Y)\, W$	p_X
(5) Market clearance for Y	$Y \geq e_{p_Y}(p_X, p_Y)\, W$	p_Y
(6) Market clearance for W	$W \geq I / p_W$	p_W
(7) Market clearance for L	$L \geq dl_X\, X + dl_Y\, Y$	p_L
(8) Market clearance for K	$K \geq dk_X\, X + dk_Y\, Y$	p_K
(9) Income balance	$I = p_L L + p_K K$	I

Applied General Equilibrium Modeling

Consider a stylized economy with the following properties:

(i) Agents: two producers and one representative household

(ii) Behavioral rules:
Producers max. profit subject to technological constraints
Consumers maximize utility subject to budget constraint

(iii) Market structure: perfectly competitive

(iv) Technologies, preferences and resource endowments:
Two factors: capital K and labour L
Cobb-Douglas production functions for goods X and Y with inputs K & L
Cobb-Douglas utility function in goods X and Y

$$X = K^{0.75} \ L^{0.25}$$

$$Y = K^{0.25} \ L^{0.75}$$

$$U = X^{0.5} \ Y^{0.5}$$

Applied General Equilibrium Modeling

(v) Benchmark flows as follows:

	Production Sectors			Consumers	
Markets	X	Y	W	CONS	Row sum
PX	100		-100		0
PY		100	-100		0
PW			200	-200	0
PL	-25	-75		100	0
PK	-75	-25		100	0
Column sum	0	0	0	0	

Use of GAMS to model this economy in a general equilibrium setting

Applied General Equilibrium Modeling

(v) Benchmark flows as follows:

	X	Y	CONS	ROWSUM
X	100		-100	0
Y		100	-100	0
L	-40	-60		0
K	-60	-40		0
COLSUM	0	0	0	

Use of GAMS/MPSGE to model this economy in a general equilibrium setting